Thermoelectric effects in a Luttinger liquid

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Thermoelectric effects in a Luttinger liquid (LL) wire adiabatically connected to the leads of noninteracting electrons are considered. For a multichannel LL a staircase-like behavior of the thermal conductance as a function of chemical potential is found. The thermopower for a LL wire with an impurity is evaluated for two cases: (i) LL constriction, and (ii) infinite LL wire. We show that the thermopower is described a Mott-like formula renormalized by an interaction-dependent factor. For an infinite LL the renormalization factor decreases with increase of the interaction. However, for a realistic situation, when a LL wire is connected to the leads of noninteracting electrons (LL constriction), the repulsive electron-electron interaction enhances the thermopower. A nonlinear Peltier effect in a LL is briefly discussed.
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1. INTRODUCTION

Charge and heat transport through a narrow wire whose width is comparable to the electron Fermi wavelength occur via a finite number of transport channels associated with quantization of the electron's transverse momentum in the wire. Furthermore, at low temperatures the phase-breaking length, $\lambda_\rho(T)$, can exceed the length of the wire, $\lambda_\rho(T)>L$, and the electron transport becomes phase coherent. In the Landauer approach[1] to such quantum mechanical transport problems the complexity of calculating the relevant transport coefficients is reduced to a single-particle scattering problem, with the transport properties of the electrons described in terms of the probability for transmission of the electrons through the effective scattering potential represented by the wire. Indeed, this approach, whose implementation is often simpler than the use of the Kubo treatment of such problems, has proved to be most useful for the description of the transport properties of noninteracting electrons through wires (constrictions) of reduced dimensions (see reviews in Ref. 2).

It is well known that for strictly one-dimensional (1D) interacting electron systems the Fermi liquid (FL) description of the low-energy excitations does not hold. Instead, for such systems with interactions which leave the electronic spectrum gapless, the corresponding "long-wavelength" theory is that of the Luttinger liquid (LL).[3] Unlike the Fermi liquid description, where charged excitations are represented by quasiparticles (electrons and holes), electrons do not propagate in an (infinite) LL. Rather, the excitation spectrum of the LL consists of gapless bosonic excitations (charge and spin density waves); harmonic oscillations of boson fields are neutral, whereas their topological excitations carry charge and spin.

Since the LL and the FL have qualitatively different excitation spectra, the transport properties of LLs have been the subject of theoretical interest, and it was shown rather early[4] that the electric conductance G of an impurity-free infinite LL depends on the interelectron interaction, i.e., $G=gG_0$, where $G_0=e^2/h$ is the quantum of conductance and g is the dimensionless electron–electron interaction parameter of the LL. Subsequent intensive investigations pertaining to transport properties of LLs were triggered by the studies of Kane and Fisher[5] and of Glazman *et al.*,[6] who considered the transport of charge through a local impurity in the LL, finding that for repulsive electron-electron interactions the conductance scales with temperature (at low temperatures) as a power law $G(T)\sim T^{2/g-2}$; such behavior has been reported in recent experiments.[7,8]

Heat transport in a LL was first considered in Ref. 9, where it was shown that in an infinite homogeneous LL the thermal conductance $K(T)$ is not renormalized by the interactions, i.e., $K(T)=K_0(T)=(\pi^2/3)k_B^2T/h$, while in the presence of an impurity $K(T)\sim T^3$. This result, together with the one for the electrical conductance, predicts violation of the Wiedemann-Franz law in a LL.

The above results, which were derived for effectively infinite LLs, cannot be tested directly in quantum wires connected to source and drain leads. To address this issue, the transport properties of the LL were considered for a finite 1D wire adiabatically connected to FL leads modeled by 1D reservoirs of noninteracting electrons. The results obtained for such a finite and impurity-free LL wire were found to be qualitatively different from those derived for the infinite LL.

In particular, it was shown that for finite LL wires with adiabatic contacts to the reservoirs the electric conductance is not renormalized by the interelectron interaction[10] and that the thermal conductance is significantly suppressed (for spinless electrons) for a strong repulsive interparticle interaction.[11,12] These predictions have a rather simple physical explanation. Since the electrons in the reservoirs are taken as noninteracting particles, one could use the Landauer approach for calculation of the electric and thermal conductances. For an adiabatic LL constriction the electrons are not backscattered by the confining potential of the wire, and consequently charge is transmitted through the wire with unit probability. Therefore, the electric conductance of a LL constriction coincides with the conductance of a single-channel quantum point contact. In contrast, heat is transported in the LL by plasmons (charge-entropy separation)[11] which, for strong interactions, are significantly backscattered at the ''transition'' region between the LL wire and the FL reservoirs, and consequently heat transport is suppressed.

The aforementioned studies dealt with spinless electrons and a single-channel LL. However, in many real situations the quantum wires may support several 1D (transport) channels, and currently thermoelectric effects in LLs remain largely unexplored. In this context, we remark that it has been noted[13] that the thermopower of a Hubbard chain, in the vicinity of a Mott-Hubbard phase transition to a dielectric phase, can be calculated using the Mott formula (see, e.g., Ref. 14) for noninteracting fermions. This observation has been exploited[15] in a derivation of the thermopower of a homogeneous infinite Hubbard chain in the limits when the Hubbard model can be mapped onto a model of spinless Dirac fermions.

In light of the above, we report here on studies of heat transport through a multichannel LL constriction connected to Fermi liquid leads, as well as investigations of the thermopower (Seebeck) and Peltier effect in a LL wire (Fig. 1).

First, we study heat transport through a multichannel LL constriction. In this case the thermal conductance as a function of the chemical potential μ demonstrates a staircase-like behavior. We show that at low temperatures $T \ll T_0 \simeq \hbar v_0/L$ (v_0 is the characteristic velocity, which is determined by the strength of the confining potential, and L is the length of the LL wire) the steps in the conductance $K(\mu)$ are practically unaffected by electron-electron interactions. On the other hand, strong interactions suppress the heat conduction at temperatures $T \sim T_0$; however, the steps are pronounced even in this high-temperature region. Subsequently, we evaluate the thermopower for a finite LL wire connected to FL leads. In this case a simple physical approach to the problem was used. The finite LL wire is modeled by an effective transmission coefficient which in the Landauer-Buttiker approach determines the charge and heat transport between the leads. We predict that the thermopower of a LL with an impurity is described by a Mott-like formula—it depends linearly on the temperature and is proportional to the logarithmic derivative of the bare (unrenormalized by the electron-electron interactions) electric conductance. At low temperatures $k_B T \ll \Delta_L \simeq \hbar s/L$ (L is the length of the LL wire, and s is the plasmon velocity) the thermopower is not renormalized by the electron-electron interactions, and it is described by the well-known formula for the thermopower S_0 for a system of noninteracting electrons (see, e.g., Ref. 14). At temperatures $k_B T \gg \Delta_L$ the interaction renormalizes the thermopower, and consequently for a strong interaction $S_{LL} \sim S_0/g^2 \gg S_0$. The renormalization factor is different for spinless and spin-1/2 electrons, and the enhancement of the thermopower is more pronounced for spinless particles.

Next, we calculate the thermopower for an infinite LL. Although the situation when the effects of the leads are excluded appears somewhat artificial from the experimental point of view, it is useful to elaborate this problem by a powerful LL calculation technique. In particular, we note that the transport properties of 1D interacting electrons have been studied mostly for an infinite LL, and thus the evaluation of the thermopower for this case represents an interesting and important theoretical problem. We show that for an infinite LL wire with an impurity the thermopower is described by the Mott formula, S_0, multiplicatively renormalized by the electron-electron interaction.

For an infinite LL the renormalization factor decreases with increase of the interaction, $S(g \ll 1) \sim g S_0$. This result does not contradict our previous claim, since the two problems under study (infinite LL wire and LL wire adiabatically connected to metallic leads) are not identical. In particular, the driving voltage which enters the definition of the thermopower is different for the two cases in question. For an infinite LL it is the voltage drop V across the impurity. In the case of the LL constriction the bias voltage U is defined as the difference of the chemical potentials of the leads, $U = \Delta\mu/e$. It has been shown[16] that for a strong impurity (weak tunneling) $V = g^2 U$. Thus the thermopower of a LL wire, when expressed in terms of U, is enhanced by interaction. This derivation supports our finding that the strong interelectron interaction strongly enhances the thermopower of a LL with an impurity.

It is well known (see, e.g., Ref. 14) that in the linear-response regime the Peltier effect is determined by the same thermoelectric coefficient as the Seebeck effect. However, in the nonlinear regime the Onsager symmetry relations between the transport coefficients cease to be valid, and the Peltier coefficient for $eV \gg k_B T$ (V is the bias voltage) describes an independent thermoelectric phenomenon. We evaluate the nonlinear Peltier coefficient for an impurity-containing LL wire connected to leads. The phenomenological approach, when the finite LL wire is modeled by an effective transmission coefficient, does not predict the renormalization of the nonlinear differential Peltier coefficient by the interaction.

The paper is organized as follows. In Sec. 2 the thermal

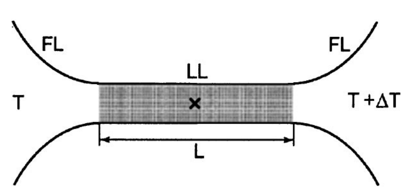

FIG. 1. Schematic of a Luttinger liquid (LL) nanowire of length L, connected to Fermi liquid (FL) reservoirs that are kept at different temperatures. The impurity (scattering potential, denoted by X) is placed in the middle of the LL wire.

conductance of a multichannel LL is studied. In Sec. 3 the thermopower of a LL constriction with an impurity is evaluated in a phenomenological approach. In Sec. 4 bosonization technique in conjunction with a tunneling Hamiltonian method is used for a calculation of the thermopower of an infinite LL. In Sec. 5 we investigate the Peltier effect in a Luttinger liquid. The main results are summarized in Sec. 6.

2. INTERACTION-ENHANCED STAIRCASE BEHAVIOR OF THE THERMAL CONDUCTANCE

To calculate the thermal conductance of a multichannel LL wire adiabatically connected to 2D reservoirs of noninteracting electrons we will use the multimode LL model developed in Ref. 16. The Hamiltonian of the model in the boson representation takes the form

$$H=\sum_{j=1}^{N}\int dx\left[\frac{p_j^2(x)}{2mn_j}+\frac{mn_j}{2}v_j^2(u_j')^2\right]+\frac{U_0}{2}\sum_{i,j=1}^{N}n_in_j\int dx f_L(x)u_i'(x)u_j'(x), \qquad (1)$$

where $u_j(x)$ is the displacement operator of the jth mode; $u_j'\equiv\partial u_j/\partial x$; p_j is the conjugate momentum, with $[u_i(x),p_j(y)]=i\hbar\,\delta_{ij}\delta(x-y)$; n_j is the number density of the electrons in the jth mode and $v_j=\pi\hbar n_j/m$ is the corresponding Fermi velocity, and U_0 determines the strength of the electron-electron interaction, which is assumed to be local: $U(x-y)=U_0\delta(x-y)$. We introduced into the Hamiltonian in Eq. (1) a smooth function $f_L(x)$ that restricts the electron-electron interaction to a finite region of length L. The electron reservoirs are modeled as 1D N-channel Fermi gases and they are represented, in the boson form, by the noninteracting part of the Hamiltonian.

The Hamiltonian in Eq. (1) is quadratic and can be easily diagonalized. In diagonal form it describes N noninteracting ‘‘bosonic’’ modes with velocities s_n $(n=1,...,N)$ which are adiabatically transformed into N modes with velocities v_n $(n=1,...,N)$. The latter modes correspond to the N noninteracting electron channels in the leads. The plasmon velocities s_n are determined by the equation[17]

$$\sum_{n=1}^{N}\frac{v_n}{s^2-v_n^2}=\frac{\pi\hbar}{U_0}. \qquad (2)$$

For a two-channel $(N=2)$ case the above equation can be easily solved, yielding

$$s_{1(2)}=\sqrt{\frac{1}{2}(u_1^2+u_2^2)\pm\frac{1}{2}\sqrt{(u_1^2-u_2^2)^2+(2U_0/\pi\hbar)^2v_1v_2}}, \qquad (3)$$

where

$$u_{1(2)}=v_{1(2)}\sqrt{1+U_0/(\pi\hbar v_{1(2)})}. \qquad (4)$$

In the limit of strong interelectron repulsion, i.e., $U_0\gg\pi\hbar v_{1(2)}$, the interaction parameters of the two-channel LL, defined as $g_n=v_n/s_n$, take the form $(v_1\geqslant v_2)$

$$g_1\equiv\frac{v_1}{s_1}\simeq\left(\frac{\pi\hbar v_1}{U_0}\frac{v_1}{v_1^2+v_2^2}\right)^{1/2}\ll 1;$$

$$g_2\equiv\frac{v_2}{s_2}\simeq\sqrt{v_2/v_1}\leqslant 1. \qquad (5)$$

Note that for spin-1/2 interacting electrons the Hamiltonian of a single channel LL is given by Eq. (1) with $N=2$ and $v_1=v_2=v$. In this case the velocity of the ‘‘spin’’ mode $s_2=v$ is not renormalized by the interaction, i.e., $g_s=1$. In the following we will see that ‘‘spin’’ channels offer ‘‘easy pathways’’ for heat transport through a LL constriction.

In the absence of electron backscattering (see discussion below) the plasmon modes are noninteracting. Consequently, the Landauer approach[1] can be used for calculation of the thermal conductance. The corresponding expression reads[11,12]

$$K(T)=\frac{1}{Th}\sum_{n=1}^{N}\int_0^\infty d\varepsilon\,\varepsilon^2\left(-\frac{\partial f_B}{\partial\varepsilon}\right)t_n(\varepsilon), \qquad (6)$$

where $f_B\equiv[\exp(\varepsilon/k_BT)-1]^{-1}$ is the Bose-Einstein distribution function of the plasmons, and $t_n(\varepsilon)$ is the probability of plasmon transmission through the nth mode of the LL. As we have said, we assume here that the contacts of the LL to the Fermi liquid reservoirs are adiabatic, which means that there is no backscattering of charged excitations in the LL. Formally Eq. (6) represents the thermal conductance of a purely bosonic noninteracting system.[18] As was shown in Refs. 11 and 12, this formula also applies to an adiabatic (no electron backscattering) LL wire, where the heat is transported by bosonic excitations (plasmons), whose dynamics, in the absence of local scatterers, is described by a quadratic Hamiltonian. These considerations lead one to conclude that Eq. (6) yields the exact thermal conductance of a LL wire in the absence of impurities. However, the plasmons could be backscattered by the ‘‘transition region’’ between the LL and the FL reservoirs. Since the width d of the transition regions obeys $\lambda_F\ll d\ll L$, we can model them as zero-width boundaries located at $x=0$ and $x=L$. Consequently, the mismatch of the plasmon velocities at the boundaries will cause strong backscattering of the plasmons. Thus the transmission coefficient $t_n(\varepsilon)$ in Eq. (6) can be obtained by taking the function $f_L(x)$ in Eq. (1) to be of the form $f_L(x)=\theta(x)\theta(L-x)$ [where $\theta(x)$ is the Heaviside step function] and matching the wave functions of the plasmons at the boundaries. Since there is no channel mixing, $t_n(\varepsilon)$ takes a form analogous to that calculated in Ref. 12:

$$t_n(\varepsilon)=\left[\cos^2\left(\frac{\varepsilon}{\Delta_n}\right)+\frac{1}{4}\left(g_n+\frac{1}{g_n}\right)^2\sin^2\left(\frac{\varepsilon}{\Delta_n}\right)\right]^{-1}, \qquad (7)$$

where $\Delta_n\equiv\hbar s_n/L$ is the characteristic energy scale for the finite LL wire, and the plasmon velocities s_n $(n=1,...,N)$ are determined by Eq. (2). Note that for spin-1/2 electrons the ‘‘spin’’ mode is not renormalized by interaction, and the corresponding correlation parameters $g_n^{(s)}=1$ $(n=1,...,N/2)$; i.e., for the ‘‘spin channels’’ one has $t_n^{(s)}=1$, and the heat transport associated with spin density wave excitations is not affected by the electron-electron interaction.

The expressions given in Eqs. (2), (5), and (7) generalize the problem of heat transport through a single-mode spinless LL[11,12] to a multichannel LL. Now the Fermi velocities v_n depend both on the chemical potential μ and the "transverse" quantum number n which characterizes the quantization of the transverse electron momentum. For a parabolic confining potential $U_c(y)=m\Omega^2y^2/2$ the corresponding transverse energy takes the values $E_n^{\perp}=\hbar\Omega(n-1/2)$ ($n=1,...,N$), and the Fermi velocity of the nth mode is given by

$$v_n=v_0\theta\left(\frac{\mu}{\hbar\Omega}+\frac{1}{2}-n\right)\left(\frac{\mu}{\hbar\Omega}+\frac{1}{2}-n\right)^{1/2}, \tag{8}$$

where $v_0=\sqrt{2\hbar\Omega/m}$. The appearance of the step function in the definition of the Fermi velocities of the multimode LL results in a staircase behavior of the electric $G(\mu)$ and thermal $K(\mu)$ conductances as functions of the chemical potential μ.

An important comment concerning Eqs. (6)–(8) is warranted here. Note that Eq. (7) is an exact result for noninteracting plasmon excitations—that is, when the electrons are not backscattered by the confining potential in the LL constriction. Such a condition is fulfilled at low temperatures and for chemical potentials satisfying $\mu\neq\hbar\Omega(n-1/2)$. In the vicinity of $\mu=\hbar\Omega(n-1/2)$ an additional electron mode is converted from an evanescent to a propagating mode. This implies that upon reaching the threshold μ for entrance into the contact, the character of the corresponding mode changes, and in doing so the mode is strongly influenced by the confining potential. Consequently, at such threshold values of the chemical potential the assumption of adiabaticity of the LL constriction fails, and in calculating the thermal conductance the contribution due to electron transport needs to be considered. However, it is well known that the transport of charge through a local (of the order of λ_F) potential in a LL is strongly suppressed due to plasmon renormalization of the bare scattering potential,[9] implying that for sufficiently long wires and for strong electron-electron repulsion the contribution of electron transport to the thermal conductance $K(T)$ is small and can be neglected. Therefore, we conclude that under such circumstances Eq. (6) is valid for practically all values of the chemical potential except at the very beginning of the steps. We note that at low temperatures, $T\ll\hbar v_0/L$, the staircase-like behavior of the thermal conductance is practically unaffected by electron-electron interaction (Fig. 2a). At high temperatures $T\gg\hbar v_0/L$ the thermal conductance, although being suppressed in the case of strong interaction,[11,12] still demonstrates a clear staircase behavior as a function of chemical potential (Fig. 2b).

3. IMPURITY-INDUCED THERMOPOWER IN A LUTTINGER-LIQUID CONSTRICTION

The thermopower is a measure of the capability of a system of charged particles to generate an electromotive force when a temperature gradient is applied across the system. In the linear-response regime it can be represented as a ratio of transport coefficients,

$$S(T,\mu)=-\frac{L(T,\mu)}{G(T,\mu)}, \tag{9}$$

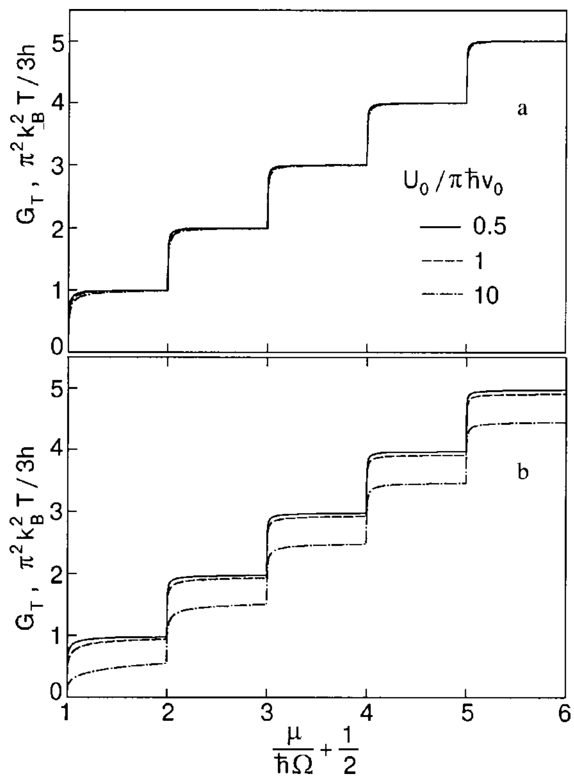

FIG. 2. The thermal conductance, in units of $\pi^2k_B^2T/3h$, plotted as a function of the dimensionless chemical potential $\mu/(\hbar\Omega)+1/2$ for several values of the strength of the electron-electron interaction $U_0/(\pi\hbar v_F)$. In (a) the temperature was taken to be $\tilde{T}=k_BTL/(\hbar v_0)=0.1$, and in (b) $\tilde{T}=10$.

where G is the electric conductance and L is the cross-transport coefficient which connects the electric current to the temperature difference for noninteracting particles. These coefficients can be calculated using a formalism developed in Ref. 19 and adapted in Ref. 18 to the Landauer scheme.[1] In this approach the transport coefficients are expressed in terms of the transmission probability $t_j(\varepsilon)$ for an electron to arrive at the drain electrode in the jth channel as

$$G(T,\mu)=G_0\sum_{j=1}^{N}\int_0^{\infty}d\varepsilon\left(-\frac{\partial f_F}{\partial\varepsilon}\right)t_j(\varepsilon), \tag{10}$$

and

$$L(T,\mu)=G_0\frac{k_B}{h}\sum_{j=1}^{N}\int_0^{\infty}d\varepsilon\left(-\frac{\partial f_F}{\partial\varepsilon}\right)\frac{\varepsilon-\mu}{k_BT}t_j(\varepsilon). \tag{11}$$

Here G_0 is the conductance quantum and $f_F(\varepsilon-\mu)$ is the Fermi-Dirac distribution function of the electrons in the leads.

Equations (10) and (11) cannot be applied to an infinite LL, where electrons are not propagating particles and the conventional scattering problem is "ill-posed." A general approach for calculating transport coefficients in a system of strongly interacting particles is the Kubo formalism, and a recent publication where it was used for calculation of the thermopower for a Hubbard chain can be found in Ref. 15. As may be seen from that study, with the Kubo approach it is difficult to calculate the thermopower in the whole range of external parameters (temperature, interaction strength, density of particles, etc.), and indeed the final analytical expressions for the desired quantities were derived[13,15] only in the limits when the Hubbard model can be mapped to a model of noninteracting fermions, for which a Mott-type expression for the thermopower could be used.

To obtain thermopower results pertaining specifically to the transport properties of systems of strongly interacting electrons, and to consider quantum-wires thermoelectric effects which could be tested in experiments, we choose to invoke at first certain simplified (yet physically reliable) models of strongly interacting electron systems. Such physical models of charge transport in LLs of strongly, as well as weakly, interacting electrons were proposed in Refs. 6 and 20 and were shown to yield the same results as those obtained from more conventional (and rigorous) treatments of LL effects,[5,21] through the use of Landauer-like expressions for estimating the dependence of the conductance on the temperature and on the bias voltage. In this Section and in Sec. 5 we use such a phenomenological approach (see also Ref. 22) for studying the Seebeck and Peltier effects in multichannel LLs.

When a LL is connected to FL reservoirs with given temperatures and chemical potentials one could make use of Eqs. (10) and (11), with $t_j(\varepsilon)$ now regarded as the probability of transmission of the electrons (in the jth channel) through the effective potential barrier formed by the LL piece of the wire. For a wire which is adiabatically connected to the leads the transmission coefficient is unity as long as we neglect the backscattering of electrons by the confining potential. For a perfect wire the backscattering effect is exponentially small for practically all values of the chemical potential, except at the narrow regions in the vicinity of conductance jumps (steps) where an additional mode is converted from an evanescent to a strongly propagating mode. Such a physical picture results in a staircase-like behavior of the conductance as a function of the chemical potential and is often modeled by abrupt jumps of the electron transmission coefficient from zero (reflected mode) to one (transmitted mode). This model is too simplified for real quantum point contacts, where the specific features of the confining potential could be important for a correct description of the transition region between the conduction plateaus. However, for strongly interacting electrons this simple model, which does not depend on the details of the bare scattering potential, could be a correct approximation. Indeed, the transmission of electrons through a long but finite LL is determined by an effective scattering potential that includes the effects of electron-electron interactions. This potential for sufficiently long wires and for temperatures $k_BT \ll E_F$ quenches all modes whose bare transmission coefficients t_0 are not very close to unity (see the corresponding discussion in Ref. 22). Since according to Eqs. (9)–(11) the thermopower $S(T,\mu) \propto \partial G/\partial\mu$ we observe that for a multimode LL constriction the thermopower vanishes on the conductance plateaus and it peaks at the conduction steps (that is, at the transition regions from one conductance plateau to the next). The qualitative distinction of the thermopower in a LL from that evaluated for noninteracting electrons[23,24] lies in the shape of the thermopower peaks. For strongly interacting electrons a simple approximation in which the (now effective) transmission coefficient is modeled by a Heaviside step function could be a quite reliable procedure. Then the temperature behavior of the peaks will be universal (it will not depend on the concrete shape of the confining potential). To make more-definite predictions we need to evaluate the thermopower for a quantum wire with a single impurity.

Since it is known that in the presence of an impurity the conductance of a LL is strongly suppressed, one may naively expect that the thermopower $S \propto \partial G/\partial\mu$ will also be strongly suppressed in such a wire. However, as we show below, that is not the case. Instead, we find that for strong (repulsive) electron-electron interactions the impurity-induced thermopower of a LL is significantly enhanced in comparison with the thermopower of a system of noninteracting particles.

To calculate the thermopower of a finite-length LL in the presence of a local impurity (which we place for simplicity at the middle of the constriction) we will model the effective transmission coefficient as

$$t^{\mathrm{eff}}(\varepsilon)=t_0(\varepsilon)\left(\frac{\Delta_L}{\Lambda}\right)^{\alpha} \quad \text{for } |\varepsilon-E_F| \ll \Delta_L, \tag{12}$$

and

$$t^{\mathrm{eff}}(\varepsilon)=t_0(\varepsilon)\left|\frac{\varepsilon-E_F}{\Lambda}\right|^{\alpha} \quad \text{for } |\varepsilon-E_F| \gg \Delta_L. \tag{13}$$

Here $t_0(\varepsilon)\ll 1$ is the bare transmission coefficient determined by the unrenormalized scattering potential (we restrict ourselves to a single-mode LL); $\Delta_L=\hbar s/L$ is the characteristic low-energy scale (s is the plasmon velocity), and Λ is the cutoff energy, which for a purely 1D LL is of the order of the Fermi energy E_F. The exponent α depends on the electron-electron interaction strength and is different for spinless and spin-1/2 electrons:[17]

$$\alpha=2\left(\frac{1}{g}-1\right); \quad g=\left(1+\frac{U_0}{\pi\hbar v_F}\right)^{-1/2} \quad \text{for } s=0, \tag{14}$$

and

$$\alpha=\frac{2}{g_s}-1; \quad g_s=2\left(1+\frac{2U_0}{\pi\hbar v_F}\right)^{-1/2} \quad \text{for } s=1/2. \tag{15}$$

The transmission probability t^{eff} in Eq. (12) results in an expression for the linear conductance which coincides (up to an irrelevant numerical constant) with that obtained in Ref. 25 via a renormalization group calculation. In fact, the same expression has been used[6] for estimation of the temperature dependence of the LL conductance in the limit of strong interaction ($g\ll 1$); this is also the limit of interest to us, since for weak interactions LL effects would be much weaker.

The bare transmission is commonly assumed to be a smooth function of the energy around E_F, i.e.,

$$t_0(\varepsilon)\simeq t_0(E_F)+(\varepsilon-E_F)\left(\frac{\partial t_0}{\partial\varepsilon}\right)_{\varepsilon=E_F}. \tag{16}$$

With this form, Eqs. (10) and (11) yield

$$G_{LL}(T)=G_0 t_0(E_F)$$
$$\times\begin{cases}\left(\dfrac{\Delta_L}{\Lambda}\right)^{\alpha}, & k_BT\ll\Delta_L,\\ 2(1-2^{1-\alpha})\Gamma(1+\alpha)\zeta(\alpha)\left(\dfrac{k_BT}{\Lambda}\right)^{\alpha}, & \Delta_L\leq k_BT\ll\Lambda,\end{cases}\tag{17}$$

and

$$L_{LL}(T)=G_0\left(\frac{\pi^2k_B^2T}{3e}\right)t_0'(E_F)$$
$$\times\begin{cases}\left(\dfrac{\Delta_L}{\Lambda}\right)^{\alpha}, & k_BT\ll\Delta_L,\\ \dfrac{6}{\pi^2}(1-2^{-1-\alpha})\Gamma(3+\alpha)\zeta(2+\alpha)\left(\dfrac{k_BT}{\Lambda}\right)^{\alpha}, & k_BT\geq\Delta_L,\end{cases}\tag{18}$$

where $\Gamma(x)$ and $\zeta(x)$ are the gamma function and the Riemann zeta function, respectively.

From Eqs. (9), (17), and (18) we conclude that at low temperatures $k_BT\ll\Delta_L$ the thermopower of a LL constriction with an impurity is not renormalized by the interelectron interactions. Instead it is described by a Mott-type formula for noninteracting electrons,[24]

$$S_0(T)\simeq-\frac{\pi^2}{3}\frac{k_B^2}{e}\left(\frac{\partial\ln G^0(\varepsilon)}{\partial\varepsilon}\right)_{\varepsilon=E_F},\tag{19}$$

where $G^0(\varepsilon)$ is the corresponding (bare) conductance of the noninteracting electrons. This finding is not surprising, since at $k_BT\ll\Delta_L$ the electrons in the leads determine the transport properties of the LL constriction. However, at temperatures $k_BT\gg\Delta_L$ the thermopower, being still a linear function of temperature, undergoes a strong multiplicative renormalization:

$$S_{LL}(T\geq\Delta_L/k_B)\simeq C_s(g)S_0(T),\tag{20}$$

$$C_s(g)=\frac{3}{\pi^2}\frac{1-2^{-1-\alpha}}{1-2^{1-\alpha}}\frac{\zeta(\alpha+2)}{\zeta(\alpha)}(\alpha+1)(\alpha+2).$$

Note that unlike the electric conductance $G_{LL}(T)$ and the cross-coefficient $L_{LL}(T)$, the thermopower $S_{LL}(T)$ does not depend on the cutoff parameter, and therefore the interaction- and spin-dependent factor $C_s(g)$ cannot be absorbed into a definition of Λ.

For noninteracting electrons $C_s(g=1)=1$, and the Mott-type formula (Eq. (19)) holds (as it should) for all temperatures ($k_BT\ll E_F$). In the limit of strong interaction $U_0\gg\pi\hbar v_F$

$$C_0(g\ll1)=12\frac{U_0}{\pi^3\hbar v_F},\tag{21}$$

$$C_{1/2}(g\ll1)=6\frac{U_0}{\pi^3\hbar v_F}.\tag{22}$$

From Eqs. (20)–(22) we observe that the LL effects on the thermopower are most significant for strong interactions, $U_0\gg\pi\hbar v_F$, and that they are more pronounced for spinless particles than for spin-1/2 electrons (Fig. 3).

Since for the thermopower the interaction dependence factorizes. Equation (20) could be readily generalized for the case of wires with dilute impurities, where the average spacing between the impurities is large enough so that the impurities act incoherently. In this case the thermopower will still be described by Eq. (20) at temperatures $k_BT>\hbar s\overline{n}$, where $\overline{n}$ is the mean concentration of the impurities. An interesting example is a LL junction made of a perfect LL wire of length L connected to leads through a potential barrier at the contacts. The thermopower of such a LL junction for temperatures $k_BT\geq\Delta_L$ is described by Eqs. (19) and (20) with the total (bare) conductance $G^0=G_1^0G_2^0/(G_1^0+G_2^0)$, where G_1^0 and G_2^0 are the (bare) conductances of the contacts.

The thermopower, being the ratio of transport coefficients, is less affected by interaction than the transport coefficients themselves (Eqs. (17), (18)). It is the prefactors in the power-law dependences of $G(T)$ and $L(T)$ on the temperature that determine the dependence of the thermopower on the interaction strength. In the phenomenological approach developed above, the quantitative correctness of these coefficients cannot be proved. Therefore, we conclude that the electron-electron interaction enhances the thermopower of a LL wire, and we will attempt to find a more rigorous treatment of the problem. In the next Section we evaluate the thermopower of an infinite LL with an impurity by making use of the bosonization technique when calculating the current in the wire induced by the bias voltage and by the temperature difference.

4. THERMOPOWER OF AN INFINITE LUTTINGER LIQUID

Let us consider an infinite LL wire with a single impurity placed (for definiteness) at $x=0$ (i.e., the middle of the wire; see Fig. 1). It is known that for a LL with repulsive electron-electron interaction the charge transport through an impurity

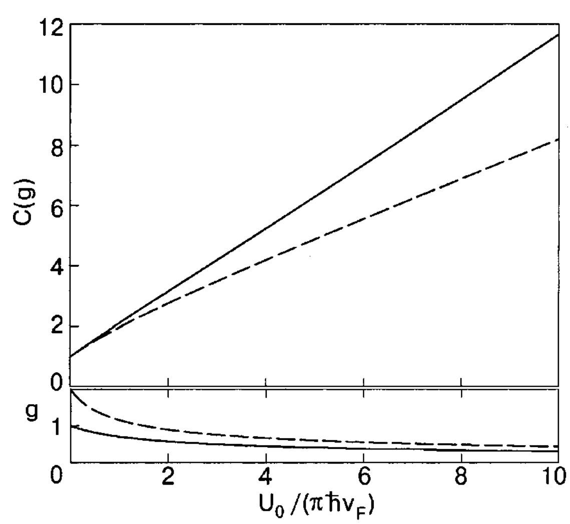

FIG. 3. The renormalization parameter $C(g)$ and the dimensionless electron interaction parameter g plotted versus the dimensionless strength of the electron-electron interaction $U_0/(\pi\hbar v_F)$ for spinless (solid line) and spin-1/2 (dashed line) electrons.

is sharply suppressed at low temperatures. Therefore, the LL is ''split'' by the impurity into two disconnected semi-infinite segments, and the charge current through the impurity can be evaluated with the use of the tunneling Hamiltonian method.

We start with the Hamiltonian

$$H=\sum_{m=1,2} H_{0,m}+H_t ,$$

where $H_{0,m}$ describes two $(m=1,2)$ identical semi-infinite parts of the LL wire. In the bosonic form it reads

$$H_{0,m}=\frac{s\hbar}{8\pi}\int dx[g(\partial_x\Theta_m)^2+g^{-1}(\partial_x\Phi_m)^2]. \tag{23}$$

Here s is the plasmon velocity, $g=v_F/s$ is the LL correlation parameter, $\Phi_m(x)$ is the displacement field, and $\Theta_m(x)$ is the field complementary to $\Phi_m(x)$, obeying the commutation relation (see, e.g., Ref. 25) $[\Theta_m(x),\Phi_m(x')]=2\pi i\delta_{mm'}\,\mathrm{sgn}(x-x')$. The tunneling Hamiltonian is

$$H_t=\int_{-\infty}^{0}dx_1\int_0^{+\infty}dx_2[\langle x_2|\hat{T}|x_1\rangle\psi_2^+(x_2)\psi_1(x_1)$$
$$+\langle x_1|\hat{T}|x_2\rangle\psi_1^+(x_1)\psi_2(x_2)], \tag{24}$$

where $\psi_m(\psi_m^+)$ is the electron annihilation (creation) operator, the index m labels two identical semi-infinite segments of the LL wire, and $\langle x_2|\hat{T}|x_1\rangle$ is the tunneling matrix element in the coordinate representation, i.e., the amplitude for the process of electron tunneling from the point x_1 to the point x_2.

Let us introduce the ''slow'' annihilation and creation operators of two types—for right- and left-moving electrons: $\psi_m(x)=\mathrm{e}^{ip_Fx}\Psi_{m,R}(x)+\mathrm{e}^{-ip_Fx}\Psi_{m,L}(x)$. At first we suggest that our contact is pointlike. Then one can simplify the tunneling Hamiltonian and write it in the form

$$H_t=\sum_{r_1,r_2}(\lambda_0\Psi_{2,r_2}^+(0)\Psi_{1,r_1}(0)+\mathrm{h.c.}), \tag{25}$$

where $\Psi_{m,r_m}(\Psi_{m,r_m}^+)$ is the operator of annihilation (creation) of an electron from the mth half of the wire (for right-moving (R) electrons $r_m=+1$, for left-moving (L) electrons $r_m=-1$).

We assume that the bare tunneling amplitude λ_0 is small. Then the tunneling rate of electrons through the barrier can be obtained to leading order from Fermi's ''golden rule.'' The total rate of electrons from the left (''1'') LL to the right (''2'') LL is of the form (see, e.g., Ref. 27)

$$\Gamma_{12}=\frac{2\pi}{\hbar}\sum_{E_1,E_2,E_1'E_2'}|\langle E_1E_2|H_t|E_1'E_2'\rangle|^2$$
$$\times P_{12}\delta\left(E_1'+E_2'-E_1-E_2-\frac{eV}{2}\right), \tag{26}$$

where P_{12} is the probability of finding the system in the state $|E_1E_2\rangle$, and V is the bias voltage. The standard evaluation (see below) of the tunnel current $J(V,T)=e[\Gamma_{12}(V,T)-\Gamma_{21}(V,T)]$ results in the well-known expression for the conductance $G(T)$ of an LL with an impurity.[5]

Let us assume now that the temperatures of the left (T_1) and right (T_2) parts of the wire are different. In this case one can expect the contribution (J_T) to the charge current induced by the temperature gradient. The Hamiltonian given by Eq. (25) with a constant bare tunneling amplitude does not allow one to evaluate this contribution. To obtain the temperature-induced current we have to take into account the finite size of the barrier. We can do it by modifying the tunneling Hamiltonian. The modified Hamiltonian includes extra terms containing the derivatives of the field operators:

$$H_t=\sum_{r_1,r_2}(\lambda_0\Psi_{2,r_2}^+(0)\Psi_{1,r_1}(0)+\mathrm{h.c.})$$
$$+\sum_{r_1,r_2}\{-i\hbar\lambda_1[r_1\Psi_{2,r_2}^+(0)\partial_x\Psi_{1,r_1}(0)$$
$$-r_2\partial_x\Psi_{2,r_2}^+(0)\Psi_{1,r_1}(0)]+\mathrm{h.c.}\}. \tag{27}$$

Here $|\lambda_1|$ is a small additional parameter $(|\lambda_1|p_F\sim|\lambda_0|)$. Notice that this form of the Hamiltonian corresponds to a tunneling amplitude which depends upon the momentum of the tunneling electron $\langle p_2|T|p_1\rangle=\lambda_0+\lambda_1r_1q_1+\lambda_1r_2q_2$, where $q_m=p_m-r_mp_F$ is the momentum of the electron toward the Fermi level.

Now the total electron current through the barrier can be written in the form

$$J=2ie|\lambda_0|^2\sum_{r_1,r_2}\int_{-\infty}^{+\infty}dt\,\sin(eVt)\langle\Psi_{2,r_2}(t)\Psi_{2,r_2}^+\rangle$$
$$\times\langle\Psi_{1,r_1}^+(t)\Psi_{1,r_1}\rangle+2ie\hbar(\lambda_0\lambda_1^*$$
$$+\lambda_0^*\lambda_1)\sum_{r_1,r_2}\int_{-\infty}^{+\infty}dt\,\cos(eVt)$$
$$\times(r_1\partial_x\langle\Psi_{1,r_1}^+(t,x)\Psi_{1,r_1}\rangle)\langle\Psi_{2,r_2}(t)\Psi_{2,r_2}^+\rangle\Big|_{x\to0}$$
$$-2ie\hbar(\lambda_0\lambda_1^*+\lambda_0^*\lambda_1)\sum_{r_1r_2}\int_{-\infty}^{+\infty}dt\,\cos(eVt)$$
$$\times(r_2\partial_x\langle\Psi_{2,r_2}(t,x)\Psi_{2,r_2}^+\rangle)\langle\Psi_{1,r_1}^+(t)\Psi_{1,r_1}\rangle\Big|_{x\to0}, \tag{28}$$

where $\langle...\rangle$ denotes the thermal average, and $\Psi_{m,r_m}(t)$ are the field operators in the Heisenberg representation, $\Psi_{m,r_m}\equiv\Psi_{m,r_m}(0)$. The correlation functions in Eq. (28) can be calculated by making use of the bosonization formula

$$\Psi_{m,r_m}(x,t)=\frac{1}{\sqrt{2\pi a}}U_{m,r_m}^+e^{-i[r_m\Phi_m(x,t)+\Theta_m(x,t)]/2}. \tag{29}$$

Here a is the cutoff parameter $(a\sim\hbar v_F/E_F)$, and U_{m,r_m}^+ is the unitary raising operator, which increases the number of electrons on the branch r_m by one particle but does not affect the bosonic excitations. We will not specify its form, since this operator enters the formulas we are studying only in the combination $UU^+=1$. Now the bosonic fields $\Phi_m(x,t)$ and $\Theta_m(x,t)$ are in the Heisenberg representation.

In our case we have to impose a boundary condition on the displacement field $\Phi_m(x)$ at $x=0$ to account for the semi-infiniteness of each segment of the LL wire, i.e.,

$$\Phi_1(0)=\Phi_2(0)=0. \tag{30}$$

Besides this, the boson fields $\Theta_m(x)$ in Eqs. (23), (29) satisfy the boundary condition

$$j(x=0)=\frac{1}{2\pi}\partial_x\Theta_m(0)=0. \tag{31}$$

The boson fields obeying the boundary conditions Eqs. (30), (31) in the momentum representation take the form

$$\Theta_m(x)=i\int_{-\infty}^{+\infty}dp\left(\frac{2s}{g\omega_p}\right)^{1/2}(b_p-b_p^+)\cos\left(\frac{\omega_p}{s}x\right),$$

$$\Phi_m(x)=\int_{-\infty}^{+\infty}dp\left(\frac{2sg}{\omega_p}\right)^{1/2}(b_p+b_p^+)\sin\left(\frac{\omega_p}{s}x\right), \tag{32}$$

where b_p and b_p^+ are the standard bosonic annihilation and creation operators ($[b_p,b_{p'}^+]=\delta_{p,p'}$); $\omega_p^0=s|p|$ is the energy of bosonic excitation with momentum p.

With the help of Eqs. (29) and (32) it is straightforward to evaluate the correlation functions. In the vicinity of the contact ($x\sim 0$) one gets the desired correlator

$$\begin{aligned}\langle\Psi_{m,r_m}^+(x,t)\Psi_{m,r_m}\rangle\simeq{}&\frac{1}{2\pi a}\\&\times\left[\frac{1}{1+iv_F\chi/a}\,\frac{\pi T_m\chi}{\sinh(\pi T_m\chi)}\right]^{1/2(1/g+r_m)}\\&\times\left[\frac{1}{1+iv_F\eta/a}\,\frac{\pi T_m\eta}{\sinh(\pi T_m\eta)}\right]^{1/2(1/g-r_m)},\end{aligned} \tag{33}$$

where $\chi=t-x/s$ and $\eta=t+x/s$. By substituting Eq. (33) into Eq. (28) we find the total electron current. In the linear-response approximation $V\rightarrow 0$, $T_1-T_2=\Delta T\rightarrow 0$, the voltage-induced ($J_V$) and temperature-induced (J_T) currents take the form

$$J_V=8i\frac{|\lambda_0|^2e^2}{(2\pi a)^2}V\int_{-\infty}^{\infty}dt\,\frac{t}{(1+iv_Ft/a)^{2/g}}\left[\frac{\tilde{T}t}{\sinh(\tilde{T}t)}\right]^{2/g}, \tag{34}$$

$$\begin{aligned}J_T={}&\frac{16i\pi e}{(2\pi a)^2}(\lambda_0\lambda_1^*+\lambda_0^*\lambda_1)\frac{k_B\Delta T}{v_F}\int_{-\infty}^{\infty}\frac{dt}{\tilde{T}t}\\&\times\left(1+i\frac{v_Ft}{a}\right)^{-2/g}\left[\frac{\tilde{T}t}{\sinh(\tilde{T}t)}\right]^{2/g}[\tilde{T}t\cosh(\tilde{T}t)-1]\\&\times\left[\tilde{T}t\cosh(\tilde{T}t)-\left(1+i\frac{v_Ft}{a}\right)^{-1}\right].\end{aligned} \tag{35}$$

Here $\tilde{T}\equiv\pi k_BT/\hbar$, where $T=(T_1+T_2)/2$ is the mean temperature.

The integrals in Eqs. (34) and (35) look very complicated. Fortunately we are interested only in the limit $a\rightarrow 0$. In this case the asymptotics of the above integrals can be easily found. Both currents $J_{V,T}$ are power-law functions of the small dimensionless parameter $\Upsilon\equiv\pi k_BTa/\hbar v_F\ll 1$. The leading terms in the asymptotics $\Upsilon\rightarrow 0$ are

$$J_V=VG(T), \tag{36}$$

$$G(T)\simeq\frac{|\lambda_0|^2e^2}{4\pi\hbar^3v_F^2}B\left(\frac{1}{2},g^{-1}\right)\left(\frac{\pi k_BTa}{\hbar v_F}\right)^{2/g-2},$$

$$J_T=k_B\Delta TL(T), \tag{37}$$

$$\begin{aligned}L(T)\simeq{}&\frac{\pi e}{2\hbar^3v_F^3}(\lambda_0\lambda_1^*+\lambda_0^*\lambda_1)k_BTB\left(\frac{3}{2},g^{-1}\right)\\&\times\left(\frac{\pi k_BTa}{\hbar v_F}\right)^{2/g-2}.\end{aligned}$$

Here $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function. Equation (36) coincides with the one found in Ref. 28. It predicts the power-law dependence of conductance on temperature. Equation (37) is a new result. From Eqs. (36) and (37) one easily gets the thermopower

$$S(g)=-\frac{k_B^2\pi^2}{e}\,\frac{B(3/2,g^{-1})}{B(1/2,g^{-1})}\,\frac{T}{|\lambda_0|^2}\,\frac{2}{v_F}(\lambda_0\lambda_1^*+\lambda_0^*\lambda_1). \tag{38}$$

For noninteracting electrons ($g=1$) Eq. (38) has to transform into the Mott formula, Eq. (19). This allows us to relate the parameters λ_0, λ_1 of the tunneling Hamiltonian to the conductance and its derivative at the Fermi energy

$$\frac{2}{v_F|\lambda_0|^2}(\lambda_0\lambda_1^*+\lambda_0^*\lambda_1)=\frac{1}{G^0}\left.\frac{\partial G^0}{\partial\varepsilon}\right|_{\varepsilon=E_F}, \tag{39}$$

where G^0 is the bare (unrenormalized by interaction) conductance. Thus, the thermopower of an infinite LL takes the form

$$S(g)=-\frac{\pi^2g}{2+g}\,\frac{k_B^2T}{e}\left.\frac{\partial\ln G^0(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=E_F}=\frac{3g}{2+g}S_0. \tag{40}$$

We showed that the electron-electron interaction in 1D systems modeled by a Luttinger liquid multiplicatively renormalizes the thermopower S_0 of the Fermi liquid. For an infinite Luttinger liquid the renormalization factor decreases with increasing interaction. At first glance this result, Eq. (40), contradicts the conclusion derived in the previous Section. Notice, however, that the two problems in question are not equivalent. It is well known, for instance, that the dependence of the conductance on the interaction strength is different for an infinite LL and for a finite LL wire connected to reservoirs of noninteracting electrons (see, e.g., Ref. 10). To relate the two problems under study we will follow the considerations presented in Ref. 16. In that paper it was shown that for a LL wire adiabatically connected to electron reservoirs the voltage drop V across the strong impurity (no electron tunneling) is connected to the voltage drop U measured on the leads by the simple relation $V=g^2U$. This formula is the manifestation of the Coulomb blockade phenomenon. Physically it is evident that in the limit of strong interaction $g^2\sim\hbar v_F/e^2\ll 1$ the shift of the chemical potentials in the leads ($\Delta\mu_L=eU$) cannot change significantly the charge densities in the LL wire "split" into two parts by a strong impurity potential. So, to relate (at least qualitatively) the thermopower $S(g)$ evaluated for infinite LL to the thermopower $S_{LL}(g)$ of a LL smoothly connected to the leads of noninteractive electrons we first of all have to replace the voltage V in Eqs. (26), (28), (34), and (36) by g^2U. Then

$S_{LL}(g) \sim S(g)/g^2 \xrightarrow{g \ll 1} S_0/g$. This means that for a real situation, when the voltage drop U is measured between the leads, the interaction enhances the thermopower. This supports our claim based on the calculations done in the phenomenological approach. Notice that there is still a discrepancy (by a factor $g^{-1} \gg 1$) between the above estimates and Eq. (40) in the limit of strong interaction. This inconsistency could be attributed to the qualitative nature of our estimations based on the phenomenological model (Sec. 3).

5. NONLINEAR PELTIER EFFECT IN A LUTTINGER LIQUID

According to the Thompson relation for the cross-coefficients of the 2×2 matrix of transport coefficients in the linear response theory, the Peltier coefficient $\Pi(T,V)$ (defined as the ratio of heat current to the electric current in the absence of a temperature gradient across the system).

$$\Pi(T,V) = \left(\frac{J_Q}{J_e}\right)_{\Delta T=0}, \tag{41}$$

obeys the relation $\Pi = -k_B TS$, where S is the thermopower. It is rather easy to verify that this relation also holds for a LL if $eV \ll k_B T$, and thus the linear Peltier coefficient in the LL can be described using Eqs. (17)–(22). In the nonlinear regime, $eV \ll k_B T$, the Onsager symmetry relations between the transport coefficients cease to be valid. For noninteracting electrons the nonlinear Peltier effect has been studied in Ref. 29, and here we remark on its behavior for a LL with an impurity.

In the Landauer-Buttiker approach the electric and heat currents between reservoirs of noninteracting electrons at fixed temperatures and chemical potentials $\mu_{1(2)}$ are given by[18,19]

$$J_e(T,V) = \frac{G_0}{e}\int_0^\infty d\varepsilon t^{\mathrm{eff}}(\varepsilon)[f_1(\varepsilon) - f_2(\varepsilon)]. \tag{42}$$

$$J_Q(T,V) = \frac{1}{h}\int_0^\infty d\varepsilon\, t^{\mathrm{eff}}(\varepsilon)(\varepsilon-\mu)[f_1(\varepsilon) - f_2(\varepsilon)], \tag{43}$$

where

$$f_{1(2)}(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \mu_{1(2)}}{k_B T}\right) + 1\right]^{-1}$$

are the distribution functions of the electrons in the reservoirs, $\mu_{1(2)} = \mu \pm eV/2$ for a symmetric LL wire, and V is the voltage drop across the wire. In the following we will use Eq. (12) (as in Sec. 3) to model the transmission probability $t^{\mathrm{eff}}(E)$ for a finite LL with an impurity placed in the middle of the wire.

Prior to proceeding with our analysis we note that the $J-V$ characteristics of a finite LL connected to FL reservoirs were studied in Refs. 30 and 22 using different approaches. In Ref. 22 the current–voltage dependence was calculated using a qualitative physical approach, similar to that employed by us in the present study, while a more rigorous treatment of charge transport through a finite LL with an impurity, based on renormalization group analysis, was elaborated in Ref. 30. Unlike the linear-response transport regime, where the above two approaches arrived at similar results, in the nonlinear regime they yield different behaviors for the current as a function of voltage at low temperatures $(T \to 0)$. Since the backscattering of the electrons by a local impurity in an infinite LL leads to a power-law dependence of the electric current on the voltage,[5] it may be expected, and is indeed found in our model, that for a finite LL this behavior would cross over to ordinary Ohmic $J-V$ behavior for $eV \ll \Delta_L$. However, the analysis given in Ref. 30 revealed the occurrence of additional oscillations of the current as a function of the bias voltage, which do not appear in our model. Underlying these oscillations is the multiple scattering of the plasmon by the impurity potential and at the boundaries of the LL, and the phase of these oscillations is sensitive to the position of the impurity. While our approximation scheme does not reveal these mesoscopic oscillations, one may expect that such fine structure in the $J-V$ characteristics would be obliterated upon averaging over the position of the impurity.

With the above assumptions, and using Eq. (12) in Eq. (42), we obtain for the differential electric conductance at $k_B T \ll eV$,

$$\frac{\partial J_e}{\partial V} = G_0 t_0(E_F)\left(\frac{\Delta_L}{\Lambda}\right)^{\alpha} \quad \text{for } eV \ll \Delta_L, \tag{44}$$

and

$$\frac{\partial J_e}{\partial V} = G_0 t_0(E_F)\left(\frac{\Delta_L}{2\Lambda}\right)^{\alpha} \quad \text{for } eV \gg \Delta_L. \tag{45}$$

In a similar fashion we obtain for the heat current at $k_B T \ll eV$

$$\frac{\partial J_Q}{\partial V} \simeq \frac{e}{h} t_0'(E_F)\left(\frac{eV}{2}\right)^2\left(\frac{\Delta_L}{\Lambda}\right)^{\alpha} \quad \text{for } eV \ll \Delta_L, \tag{46}$$

and

$$\frac{\partial J_Q}{\partial V} \simeq \frac{e}{h} t_0'(E_F)\left(\frac{eV}{2}\right)^2\left(\frac{eV}{2\Lambda}\right)^{\alpha} \quad \text{for } eV \gg \Delta_L. \tag{47}$$

From Eqs. (44)–(47) it is readily seen that within the framework of our calculations the nonlinear Peltier coefficient for a symmetric LL constriction with an impurity placed at the middle of the LL wire does not depend on the interelectron interactions, and the differential Peltier coefficient is given by (at $k_B T \ll eV$)

$$\Pi(V) \equiv \frac{\partial J_Q/\partial V}{\partial J_e/\partial V} \simeq \frac{1}{e}\left(\frac{eV}{2}\right)^2\left(\frac{\partial \ln t_0(\varepsilon)}{\partial \varepsilon}\right)_{\varepsilon=E_F}. \tag{48}$$

We remark, however, that an influence of the interelectron interactions on the Peltier coefficient may occur for asymmetric LL wires or when the aforementioned mesoscopic oscillations are included.

6. CONCLUSIONS

In this paper we have used physically motivated models to investigate the heat transport through a multichannel LL wire and also the thermopower and Peltier effect in a single-channel LL with an impurity.

(i) For a multichannel LL wire, we predict that electron-electron interactions would stabilize the staircase-like behavior of the thermal conductance $K(T,\mu)$ as a function of the chemical potential (which can be controlled through the use of a gate voltage). For strongly interacting particles the jumps in the thermal conductance at each value $\mu=\mu_n$ at which a new propagating channel is allowed to enter the constriction remain sharp even at comparatively "high" temperatures.

(ii) For a perfect (impurity-free) LL wire the thermopower (Seebeck coefficient) vanishes on the conductance plateaus and it peaks sharply at the conductance jumps. We also considered the thermopower effect for a single-channel LL constriction with an impurity placed at the middle of the constriction. For this system the Mott expression for the thermopower holds at low temperatures $k_B T \ll \Delta_L = \hbar s/L$, where s is the plasmon velocity and L is the length of the LL wire. However, at $k_B T > \Delta_L$ the thermopower is multiplicatively renormalized by the electron-electron interactions. The effect of this renormalization is predicted to be more pronounced for spinless particles than for spin-1/2 electrons. This conclusion is supported by an evaluation of the thermopower for an infinite LL with an impurity by the tunnel Hamiltonian method. The Peltier coefficient $\Pi(T,V)$ of a LL wire, in the *linear*-response regime, is determined by the thermopower, $\Pi(T) = -k_B T S_{LL}(T)$. Unlike the thermopower (Seebeck coefficient) the nonlinear Peltier coefficient is found in our model to be unaffected by the interelectron interactions, and thus it is determined by the energy dependence of the bare probability of transmission through the wire.

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